Context and summary of my past and present work

Aline Goeminne

Context

Game theory and rationality. Game theory is a branch of mathematics that models interactions between individuals (called players or agents). Von Neumann and Morgenstern are known as the fathers of game theory. They published in 1944 the book "Theory of Games and Economic Behavior". Their work is based on an axiomatization of the rational behavior of a decision maker. Such a decision maker should maximize the expected value of his utility function if he is rational. Game theory has been applied in many disciplines such as economics, biology or computer science. In my research context, I am particularly interested in games played on graphs.

Two-player zero-sum games played on graphs. Games played on graphs are used to model several situations. Two-player zero-sum games are a particular case of such games. In this context players interact together and aim at reaching antagonistic objectives. Another point of view is that one of the two players wants to achieve his objective in the worst scenario (modeled by the behavior of the antagonistic second player). In a game played on a graph, the players' different possible actions and the results of these actions are described by the game graph. An infinite path in this graph is called a play. In such a game, each player chooses a strategy: it is the way he plays given some information about the game and past actions of the other player. Following a strategy profile, i.e., a tuple of strategies (one per player), results in a particular play called the outcome. Finding how a player can ensure to achieve his objective amounts to finding a winning strategy for this player, i.e., a strategy which ensures that the player achieves his goal whatever the strategy of the opponent.

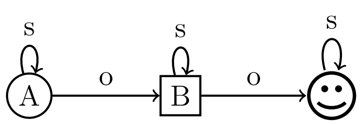

Figure 1: A game played on a graph

Ex: We illustrate these concepts on the example which is modeled by the game depicted in Figure 1. We assume that Alice and Bob are in the same room (vertex A) and Alice wants to reach another room (the smiley). Two doors have to be opened in order to achieve the Alice's objective. Alice has the key of first room (vertex A), while Bob has the key of the second (vertex B). In this example, Alice has the system role and Bob the environment role. They both have the choice to stay in the current room (edge labeled by s) or to open the door and move in the next room (edge labeled by o). In this example, Alice has no winning strategy as Bob can choose to stay infinitely in vertex B.

From two-player games to multiplayer games. This previous model is not realistic since it assumes that the two players are fully antagonistic and that there are only two players that interact together. Thus, the model evolves from two-player zero-sum games to multiplayer games where all players have their own objectives not necessarily antagonistic. In this setting, the solution concept of winning strategy is not well suited anymore. Equilibria are widely studied in multiplayer games, e.g., Nash equilibrium (NE), subgame perfect equilibrium (SPE), secure equilibrium,... Roughly speaking, an equilibrium is a contract between the players such that each player has no incentive to deviate from this contract if he assumes that the other players will follow it.

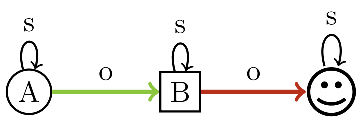

Figure 2: Nash equilibrium

Ex: We now assume that Bob also wants to reach the smiley vertex and we are looking for an NE. In Figure 2, an Alice's strategy (resp. Bob's strategy) is highlighted in green (resp. in red): they both open the door. Since both player achieves his objective by complying with these strategies, i.e., they reach the smiley vertex, they have no incentive to modify their own strategy. In particular, this means that the strategy profile depicted in Figure 2 is an NE such that all players achieve their objective.

Qualitative vs quantitative objectives. Qualitative objectives (e.g., reaching a vertex) only offer to express if a specification is satisfied or not. However, one may need to express quantitative specifications (quantitative objectives) , e.g., measuring the cost (money, time, . . .) to reach a vertex.

Computer systems and formal methods. Nowadays computer systems are more and more involved in our everyday life. These systems become increasingly complex and interact either together or with humans. Moreover, some of these reactive systems are used by humans for critical tasks such as medicine, transport, etc. For this kind of critical systems, bugs may have dramatic consequences. For this reason one wants to ensure that a system is correct and satisfies some properties. One way to do so is program testing but as E.W. Dijkstra said: "Program testing can be used to show the presence of bugs, but never to show their absence". Formal methods through concepts such as model checking and synthesis provide techniques to mathematically prove that a system is correct. Model checking allows to systematically check whether some properties hold in the system. We have to give formal descriptions of the system thanks to a mathematical model (e.g., a graph) and of the properties that the system has to satisfy (e.g., thanks to linear temporal logic (LTL) formulae). Most of our work to date has been in this context.

Games played on graphs are widely used to model this kind of considerations. Two-player zero-sum games allows to represent the interaction between a system and its environment. In this setting, the system's objective represents the property that should be satisfied and the environment acts in an antagonistic way. The underlying game is the following: the two players are the system and the environment and the vertices of the graph are all configurations in which the system can be. Finding how the system can ensure the satisfiability of the given property amonts to finding a winning strategy in this game for the system. Multiplayer games allow to model more complex situations where the system is composed by several components which have their own objective.

Past and current work

My related publications, given in the same alphabetical format as in the rest of my website, are mentioned for each topic.

Characterization and synthesis of relevant solution concepts.

With simple solution concepts.
Related publications: [BBGR18, BBG+19, BBGT19, Goe20, BGMR23] (Some have also been published as special issues.)

The existence of equilibria is widely studied and it is known that several equilibria may coexist in the same game (e.g., [1]). Nevertheless, some equilibria are more relevant than others. For example, if we consider a game in which each player aims at satisfying a qualitative objective, it is possible to have both an equilibrium in which no player satisfies his objective and another one in which each player satisfies it. In this case, one prefers the latter equilibrium.

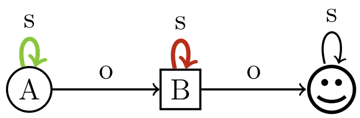

Figure 3: Non relevant Nash equilibrium

Ex: If Alice and Bob choose to stay in the current room (strategies highlighted in color in Figure 3), they do not achieve their objective as they stay in vertex A ad infinitum. However, this behavior forms an NE since Alice has no incentive to open the door as they will be stuck in vertex B and in the same way, if Alice complies with her strategy, a change in the strategy of Bob has no influence on the outcome. Obviously, the NE in Figure 2 is more relevant than this latter one.

In our work, we investigated the existence of relevant equilibria (for some relevance criteria) by providing the complexity classes of related decision problems. Our approach relies on outcome characterizations of equilibria (NEs and (weak) SPEs). In a few words, thanks to those characterizations, deciding the problems amounts to finding plays (or finite set of plays) that satisfy some good properties. This change of perspective allowed us to decide the existence of relevant equilibria. We also studied upper bounds on the required memory of relevant equilibria.

With permissive solution concepts.
Related publications: [BG23,GM25]

Even when a strategy is synthesized, its implementation may fail. This can be due to the occurrence of errors; for example, the action prescribed by the strategy may be unavailable. To address these robustness issues, the notion of strategy can be replaced by the notion of multi-strategy: a multi-strategy of a player prescribes a set of allowed possible actions when it is its turn to play. In this setting, we aim at synthesizing the most permissive multi-strategies. Intuitively, a multi- strategy is more permissive than another if the first allows more behaviors than the second. Inspired by [2] which investigates permissiveness in two-player zero-sum qualitative reachability games by introducing penalties (a quantitative way to compare multi-strategies), we investigated those notions both in (i) two-player zero-sum quantitative reachability games and (ii) in multiplayer qualitative reachability games. By studying (i), we first solved questions about two-player zero-sum multi-weighted reachability games. In this setting, the system has several quantitative reachability objectives that he wants to optimize. We ask what cost profiles can be ensured regardless of the environment's behavior. Then, we have extended those results to obtain complexity results about permissiveness in two-player quantitative reachability games. Finally, for (ii), we provided outcomes characterizations of permissive NEs and SPEs that allowed us to decide problems about the existence of relevant permissive equilibria.

Adding time into the picture.

As a first step towards more realistic models, we considered timed games that allow to model real-time specifications. In this setting, the game graph is enhanced with clocks that are variables which evolve over time. Moreover, a transition in the graph may be chosen only if clocks satisfy some constraints imposed by this transition. This kind of graphs correspond to timed automata introduced by Alur and Dill [3]. A major difficulty of this model is that its semantic can be given thanks to an infinite game graph while our previous work focused on games with finite ones.

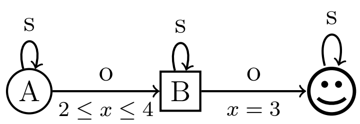

Figure 4: Timed game

Ex: Let us assume now that the door in room A (resp. in room B) can only be opened dure time 2 and 4 (resp. exactly at time 3). This situation is modeled by adding a clock, called x, and corresponding timed constraints on edges (see Figure 4). Alice can decide to wait 2.5 time units before opening the door and reaching the room B. Then, if Bon wants to reach the smiley, he is forced to wait exactly 0.5 time unit before opening the door and moving to the next room.

Towards multiplayer timed games and game bisimulations.
Related publications: [BG20]

As a first step, we studied qualitative reachability timed games and applied our previous research about the existence of relevant equilibria. Thanks to the well-known notion of bisimulation equivalence relation, we proved that under certain conditions on the game, it is possible to transform the original game graph into a finite one. The obtained game preserves enough properties to study the existence of relevant SPEs.

Timed network congestion games.
Related publications: [GMS22]

In order to study models closer to practical problems, we studied timed network congestion games. Network congestion games [4] are very popular since they have practical applications in transportation or communication networks [5]. Given a graph, the aim of each player is to reach his target from his source. Here, we consider timed network games in which the availability of edges depends on discrete time. A non-decreasing cost function is assigned to each vertex in such a way that the more players there are who stay in this vertex the more it costs them. This represents the congestion in a vertex. The semantic of this model is given by a timed (concurrent) multiplayer game in which we studied the existence of relevant NEs .

References

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