Permissive Equilibria in Multiplayer Reachability Games

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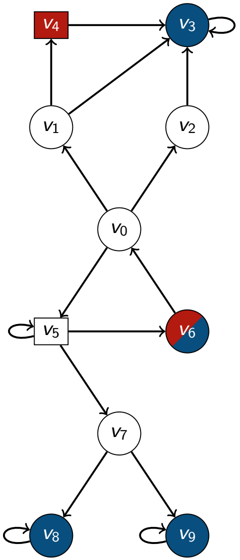
Highlights'24

Multiplayer reachability games



- A graph $G = (V, E)$
- A set of n players N, Ex: Player $\bigcirc$ and Player $\square$
- An initial vertex, Ex: v_0

Reachability objective

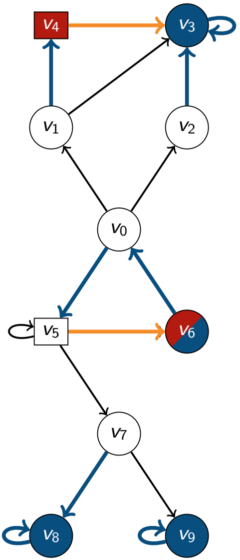
Given a target set $F_i \subseteq V$, for every play $\rho = \rho_0\rho_1 \ldots$,

$$\text{Gain}_i(\rho) = \begin{cases} 1 & \exists k \in \mathbb{N},\ \rho_k \in F_i \\ 0 & \text{otherwise} \end{cases}$$

Ex: $F_{\bigcirc} = \{v_3, v_6, v_8, v_9\}$ and $F_{\square} = \{v_4, v_6\}$

- $\text{Gain}(v_0 v_5 v_7 v_8^\omega) = (\text{Gain}_{\bigcirc}(v_0 v_5 v_7 v_8^\omega), \text{Gain}_{\square}(v_0 v_5 v_7 v_8^\omega)) = (1, 0)$

Simple strategies and Nash equilibria



- **(Simple) strategy:** $\sigma_i : V^* V_i \to V$
  Ex: $(\sigma_{\bigcirc}, \sigma_{\square})$
- **(Simple) strategy profile:** $\sigma = (\sigma_1, \ldots, \sigma_n)$
  $\rightsquigarrow \langle \sigma \rangle_{v_0}$ the **outcome**.
  Ex: $\langle \sigma_{\bigcirc}, \sigma_{\square} \rangle_{v_0} = v_0 v_5 v_7 v_8^{\omega}$.

Nash equilibrium

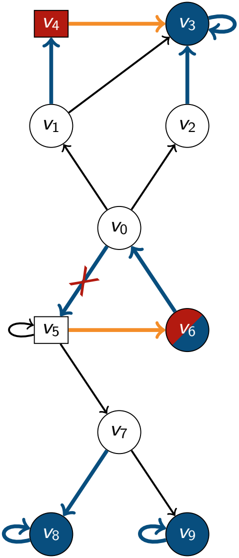
A **simple strategy profile** σ is a Nash equilibrium (NE) if **no** player has an incentive to **deviate** unilaterally.

CEx:

- $(\sigma_{\bigcirc}, \sigma_{\square})$ is **not** an NE
- $\text{Gain}(\langle \sigma_{\bigcirc}, \sigma_{\square} \rangle_{v_0}) = (1, 0)$
- $\sigma_{\square}$ is a profitable deviation
- $\text{Gain}_{\square}(\langle \sigma_{\bigcirc}, \sigma_{\square} \rangle_{v_0}) = \text{Gain}_{\square}((v_0 v_5 v_6)^{\omega}) = 1$

Ex:

- $(\sigma_{\bigcirc}, \sigma_{\square})$ is an NE.

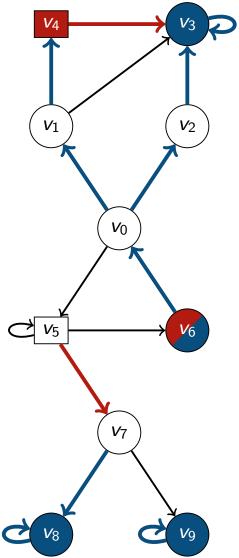


What happens if the edge (v_0, v_5) becomes **unavailable**?

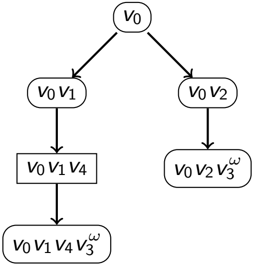
⇝ choosing (v_0, v_1) or (v_0, v_2) is also winning for Player ◯

⇝ strategies with multiple choices (**multi-strategies**)

Multi-strategies and permissive NEs

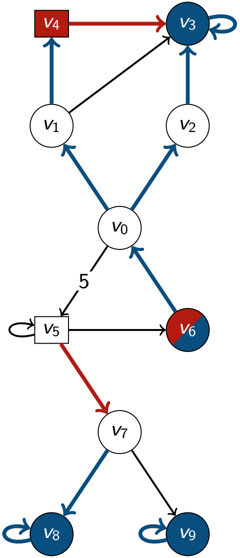


- **Multi-strategy:** $\Theta_i : V^* V_i \to \mathcal{P}(V) \setminus \{\emptyset\}$
  Ex: $(\Theta_{\bigcirc}, \Theta_{\Box})$
- **Multi-strategy profile:** $\Theta = (\Theta_1, \ldots, \Theta_n)$
  $\rightsquigarrow \langle \Theta \rangle_{v_0}$ the **set** of outcomes
  Ex: $\langle \Theta_{\bigcirc}, \Theta_{\Box} \rangle_{v_0} = \{ v_0 v_1 v_4 v_3^{\omega}, v_0 v_2 v_3^{\omega} \}$



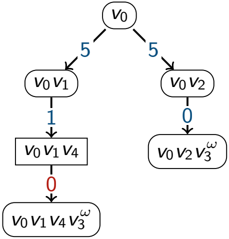
- $\rightsquigarrow$ can be seen as a tree $\mathcal{T}_{\Theta}$.

Multi-strategies and permissive NEs



How to compare two multi-strategies?

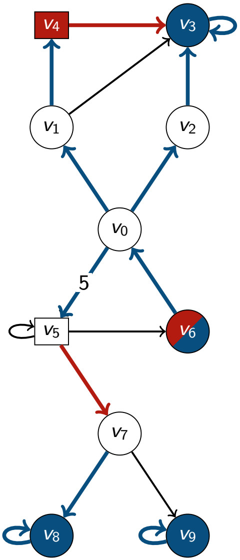
⇝ notion of **penalty**[1].



Penalties: $(6, 0)$

1. This notion of penalty is already used in the setting of two-player zero-games with reachability objectives in [BDMR09]: Measuring permissivity in finite games, P. Bouyer, M. Duflot, N. Markey and G. Renault, CONCUR'09.

Multi-strategies and permissive NEs



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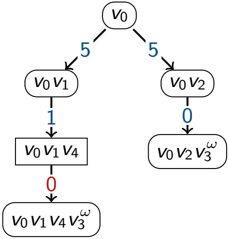

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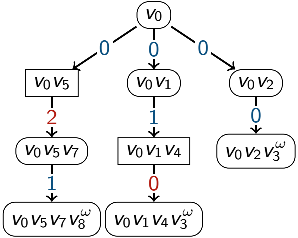

Penalties: $(1, 2)$

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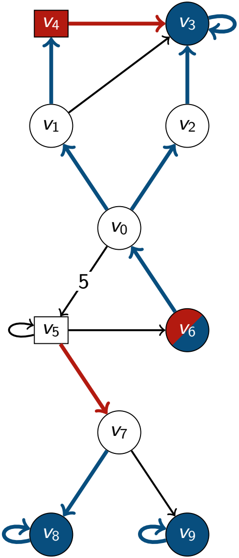
Multi-strategies and permissive NEs



- a simple strategy σ_i is **consistent** with a multi-strategy Θ_i, written $\sigma_i \lesssim \Theta_i$, if for all $hv \in V^* V_i$:

$$\sigma_i(hv) \in \Theta_i(hv).$$

Permissive Nash equilibrium

A multi-strategy profile $\Theta = (\Theta_1, \ldots, \Theta_n)$ is a **permissive NE** if for all $\sigma = (\sigma_1, \ldots, \sigma_n)$ such that, for all $1 \leq i \leq n$, $\sigma_i \lesssim \Theta_i$, σ is an NE.

Ex: $(\Theta_{\bigcirc}, \Theta_{\square})$ is a permissive NE.

Studied problems

Constrained penalty problem

Given a reachability game, an initial vertex v_0 and upper-bounds $(p_1, \ldots, p_n) \in (\mathbb{N} \cup \{+\infty\})^n$,
does there exist a multi-strategy Θ such that

- $\text{Penalty}_i(\Theta, v_0) \leq p_i$ for all $1 \leq i \leq n$.
- Θ is a permissive NE;

The constrained penalty problem can be solved in PSPACE
(if the upper-bound penalties are encoded in unary).

$\rightsquigarrow$ based on a characterization of outcomes of permissive NEs.

Conclusion

- **Permissiveness of equilibria**: **Nash equilibria** and subgame perfect equilibria;
  $\rightsquigarrow$ based on the notion of penalties.
- **Decision problems**: **constrained penalty problem**, weakly winning with constrained penalty problem and strongly winning with constrained penalty problem
  $\rightsquigarrow$ in PSPACE (if the upper-bound penalties are encoded in unary)
- **Characterization of outcomes of equilibria**: **permissive Nash equilibria** and permissive subgame perfect equilibria;
  $\rightsquigarrow$ trees have to satisfy some "good" properties.

Patricia Bouyer, Marie Duflot, Nicolas Markey, and Gabriel Renault.
Measuring permissivity in finite games.
In Mario Bravetti and Gianluigi Zavattaro, editors, CONCUR 2009, volume 5710 of LNCS, pages 196–210. Springer, 2009.