Non-Blind Strategies in Timed Network Congestion Games

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FORMATS'22

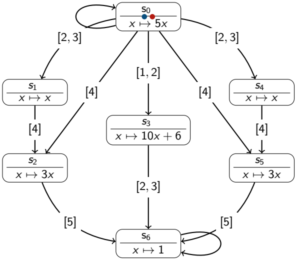
- Games in which players **share resources**: e.g., edges or locations in a **network**.

  $\rightsquigarrow$ leads to **congestion**.
- Network or **timed** network.
- Different kinds of strategies: (timed) paths vs **non-blind** strategies.
- Study of **Nash equilibria** and their **efficiency** (Social welfare, Price of Anarchy and Price of Stability).

1. Preliminaries
   - Timed network congestion game
   - Semantics as an infinite concurrent game
   - Nash equilibrium
2. Studied problems
3. Constrained existence of NEs
4. Conclusion

Preliminaries

Timed network congestion game



A timed network $\mathcal{A}$: a timed automaton

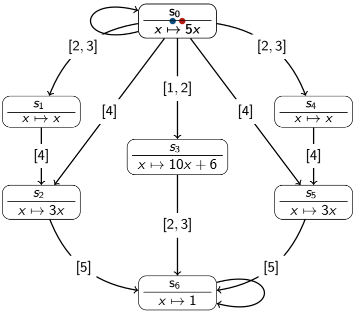
- a set of vertices (locations) V;
- a set of edges (transitions) E;
- **one** clock which is **never reset**;
- for all $e \in E$, a **guard** g_e: either True or a time interval;
  ex: $g_{s_0,s_4} = [2, 3]$.

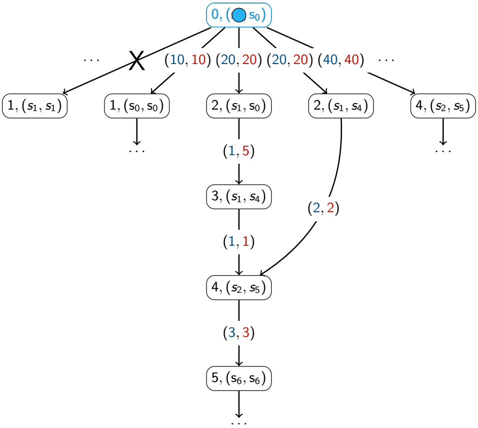
Timed Network Congestion Game (TNCG) $\mathcal{N}$

- n players (encoded in binary), $N = \{1, \ldots, n\}$;
  ex: Player 1 and Player 2;
- a timed network $\mathcal{A}$;
- for all $\mathbf{v} \in \mathbf{V}$, a non-decreasing function $L_v : N \to \mathbb{N}_0$;
  ex: $L_{s_3} : x \mapsto 10x + 6$.
- for all players $i \in N$, a **source vertex** src_i and a **target vertex** tgt_i;
  ex: $\text{src}_1 = \text{src}_2 = s_0$ and $\text{tgt}_1 = \text{tgt}_2 = s_6$.

- a **configuration** $\text{Config} = (s_1, \ldots, s_n) \in V^n$ provides the position of each player;
- a **timed configuration** $(d, \text{Config}) \in \mathbb{N} \times V^n$ provides the position of each player at a given time d.

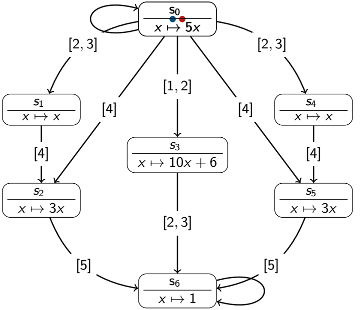
How to play in this game?

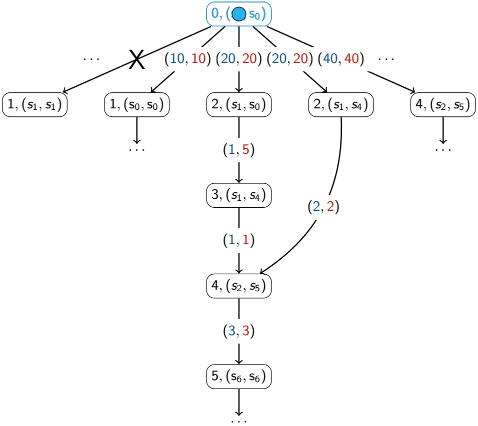




current timed config.	choice of the players	legal actions
$(0, (s_0, s_0))$		(1) absolute time in $\mathbb{N}_{>0}$ (2) a successor (1)-(2) satisfy the guards

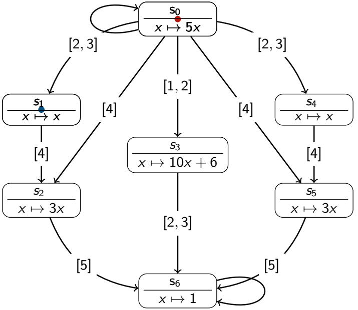
- Play: $(0, (s_0, s_0))$

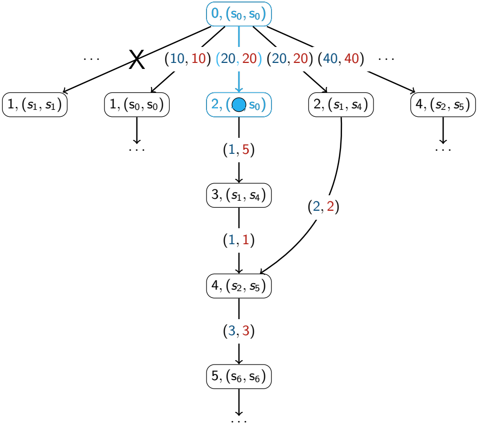




current timed config.	choice of the players	legal actions
$(0, (s_0, s_0))$	$\begin{bmatrix}(2, s_1) \\ (3, s_4)\end{bmatrix}$	(1) absolute time in $\mathbb{N}_{>0}$ (2) a successor (1)-(2) satisfy the guards

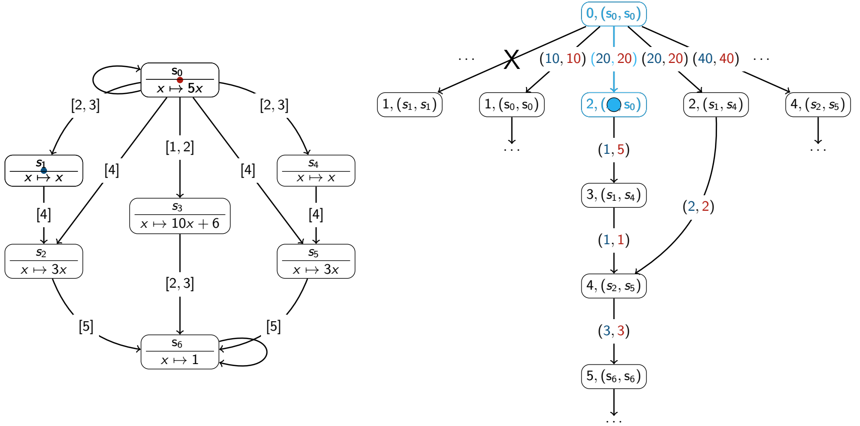
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1) \\ (3, s_4)\end{bmatrix}}$





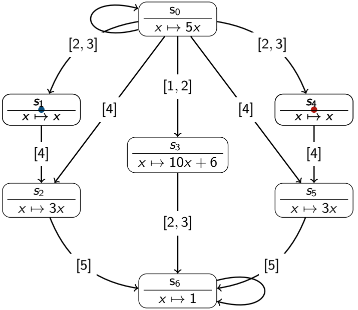
current timed config.	choice of the players	legal actions
$(2, (s_1, s_0))$		(1) absolute time in $\mathbb{N}_{>2}$ (2) a successor (1)-(2) satisfy the guards

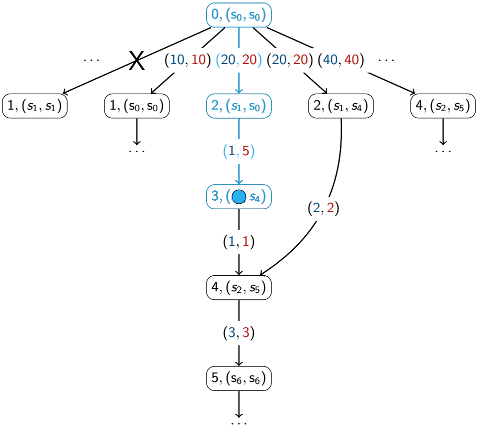
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0))$



current timed config.	choice of the players	legal actions
$(2, (s_1, s_0))$	$\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}$	(1) absolute time in $\mathbb{N}_{>2}$ (2) a successor (1)-(2) satisfy the guards

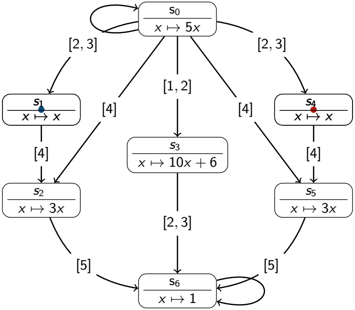
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}}$

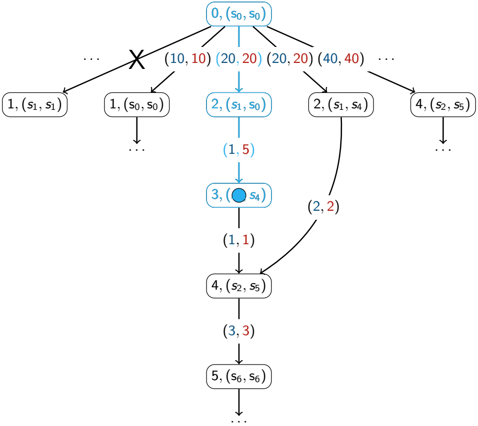




current timed config.	choice of the players	legal actions
$(3, (s_1, s_4))$		(1) absolute time in $\mathbb{N}_{>3}$ (2) a successor (1)-(2) satisfy the guards

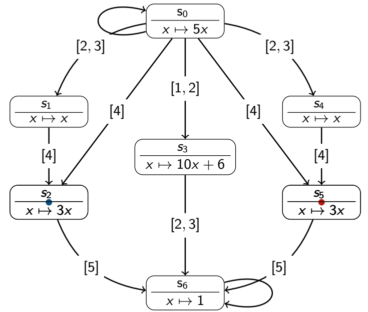
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4))$

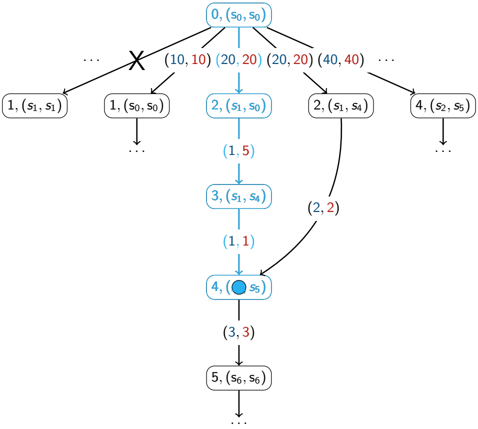




current timed config.	choice of the players	legal actions
$(3, (s_1, s_4))$	$\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}$	(1) absolute time in $\mathbb{N}_{>3}$ (2) a successor (1)-(2) satisfy the guards

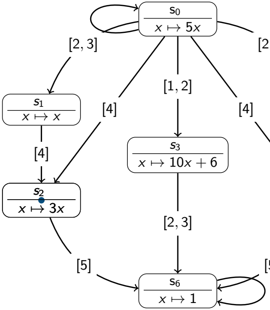
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}}$

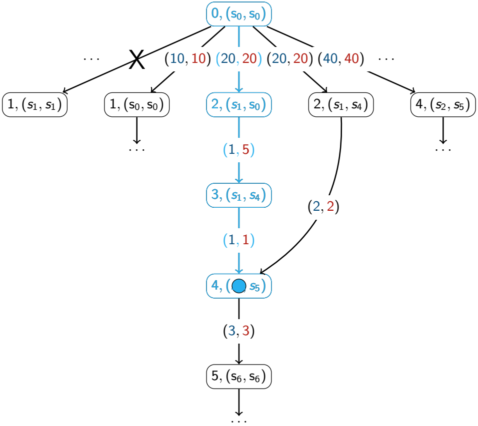




current timed config.	choice of the players	legal actions
$(4, (s_2, s_5))$		(1) absolute time in $\mathbb{N}_{>4}$ (2) a successor (1)-(2) satisfy the guards

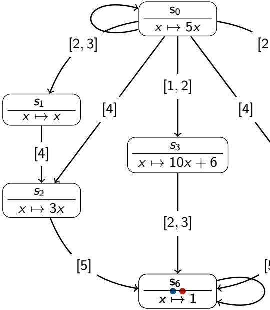
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}} (4, (s_2, s_5))$

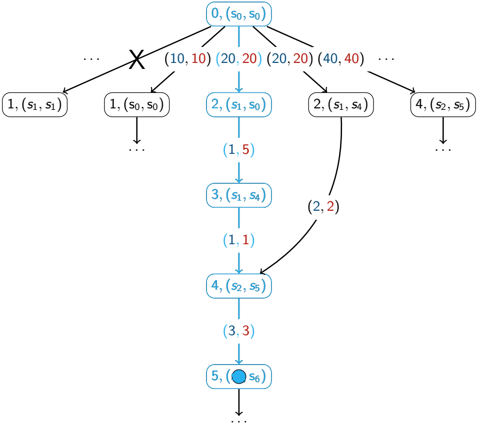




current timed config.	choice of the players	legal actions
$(4, (s_2, s_5))$	$\begin{bmatrix}(5, s_6)\\(5, s_6)\end{bmatrix}$	(1) absolute time in $\mathbb{N}_{>4}$ (2) a successor (1)-(2) satisfy the guards

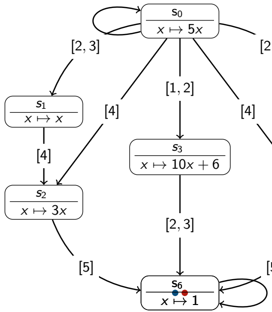
- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}} (4, (s_2, s_5)) \xrightarrow{\begin{bmatrix}(5, s_6)\\(5, s_6)\end{bmatrix}}$

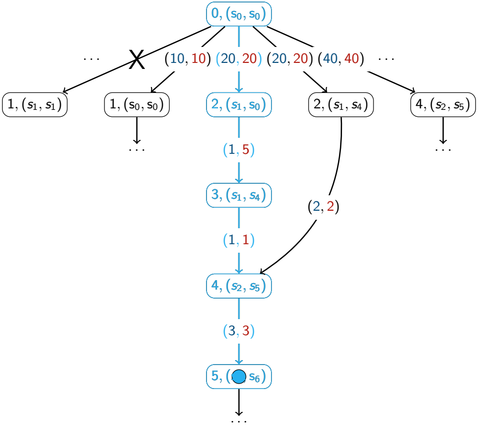




current timed config.	choice of the players	legal actions
$(5, (s_6, s_6))$	. . .	(1) absolute time in $\mathbb{N}_{>5}$ (2) a successor (1)-(2) satisfy the guards

- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}} (4, (s_2, s_5)) \xrightarrow{\begin{bmatrix}(5, s_6)\\(5, s_6)\end{bmatrix}} (5, (s_6, s_6)) \ldots$

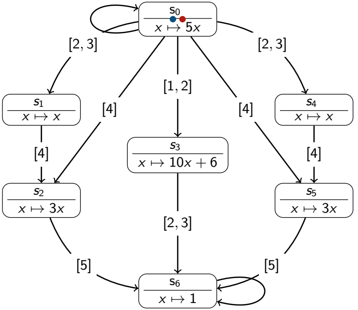




current timed config.	choice of the players	legal actions
$(5, (s_6, s_6))$	...	(1) absolute time in $\mathbb{N}_{>5}$ (2) a successor (1)-(2) satisfy the guards

- Play: $(0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}} (4, (s_2, s_5)) \xrightarrow{\begin{bmatrix}(5, s_6)\\(5, s_6)\end{bmatrix}} (5, (s_6, s_6)) \ldots$
- Cost : $(20, 20) + (1, 5) = (1, 1) + (3, 3) = (25, 29)$.

Semantics as an infinite concurrent game



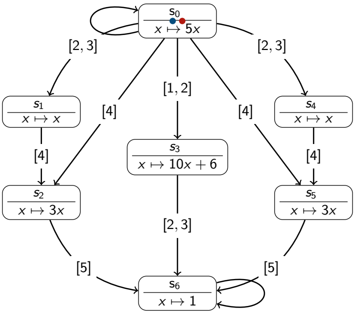
Infinite concurrent game $\mathcal{G}$

- n players;
- the set of **timed configurations** $\mathsf{TC} \subseteq \mathbb{N} \times V^n$;
- the set of **actions** $\mathsf{Act} = \mathbb{N}_0 \times V$;
- for all $i \in N$, $\mathsf{Mov}_i : \mathsf{TC} \to \mathcal{P}(\mathsf{Act})$ maps all timed config. to the set of **legal actions**
  ex: $\mathsf{Mov}_1(2, (s_0, s_4)) = \{(3, s_1), (3, s_4), (4, s_2), (4, s_5), (k, s_0) \mid k \geq 3\}$
- an **update function** $\mathsf{Up} : \mathsf{TC} \times \mathsf{Act}^n \to \mathsf{TC}$:
  ex:

$$\left((0, (s_0, s_0)), \begin{bmatrix}(2, s_1)\\(3, s_3)\end{bmatrix}\right) \mapsto (2, (s_1, s_0))$$

$$\left((0, (s_0, s_0)), \begin{bmatrix}(3, s_1)\\(3, s_3)\end{bmatrix}\right) \mapsto (3, (s_1, s_3))$$

Semantics as an infinite concurrent game



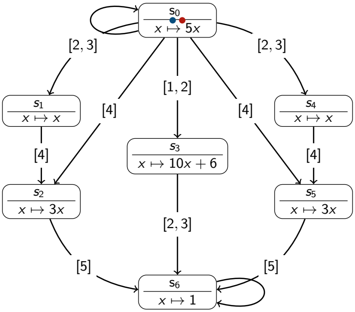
Infinite concurrent game $\mathcal{G}$

- for all $i \in N$, a weight function $w_i : \mathsf{TC} \times \mathsf{TC} \to \mathbb{N}_0$
  ex:
  $w_1, w_2 : ((0, (s_0, s_0)), (2, (s_1, s_0)) \mapsto 20$
  $w_1 : ((2, (s_1, s_0)), (3, (s_1, s_4)) \mapsto 1$
  $w_2 : ((2, (s_1, s_0)), (3, (s_1, s_4)) \mapsto 5$
- for all $i \in N$, a **target set** $\mathsf{Goal}_i = \{(d, \mathsf{Config}) \in \mathsf{TC} \mid \mathsf{Config}(i) = \mathsf{tgt}_i\}$
  ex: $(5, (s_6, s_6)) \in \mathsf{Goal}_1$.
- an **initial timed configuration** c_0;
  ex: $c_0 : (0, (s_0, s_0))$

The set of **transitions** T

$$\{(c_1, m, c_2) \in \mathsf{TC} \times \mathsf{Act}^n \times \mathsf{TC} \mid \forall i \in N,\ m_i \in \mathsf{Mov}_i(c_1) \wedge \mathsf{Up}(c_1, m) = c_2\}$$

Costs and strategies

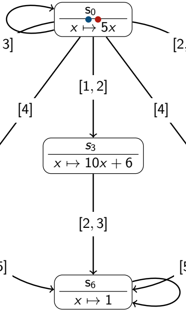


- **Computation of cost**: for each infinite path (called a **play**) ρ in $(\mathcal{G}, c_0)$, $\rho = \rho_0\rho_1 \ldots$:

$$\text{Cost}_i(\rho) = \begin{cases} \displaystyle\sum_{k=0}^{\ell-1} w_i(\rho_k, \rho_{k+1}) & \text{if } \ell \text{ is the least ind.} \\ & \text{st. } \rho_\ell \in \text{Goal}_i \\ +\infty & \text{otherwise} \end{cases}$$

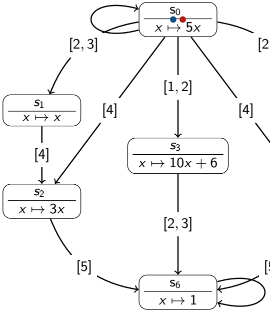
- **Strategies:** for $i \in N$, $\sigma_i : \text{Hist}_{\mathcal{G}}(c_0) \longrightarrow \mathbb{N}_0 \times V$;
- **A strategy profile:** $\sigma = (\sigma_i)_{i \in N}$;
- **The outcome** of σ from c_0: $\langle \sigma \rangle_{c_0}$;
- **The cost profile** of a play ρ: $\text{Cost}(\rho) = (\text{Cost}_i(\rho))_{i \in N}$;
- **The social welfare** of a play ρ: $\text{SW}(\rho) = \sum_{i \in N} \text{Cost}_i(\rho)$.

Examples



- Player 1 follows the timed path $(0, s_0)(2, s_1)(4, s_2)(5+k, s_6)_{k \geq 0}$;
  Player 2 follows the timed path $(0, s_0)(3, s_4)(4, s_5)(5+k, s_6)_{k \geq 0}$.
- $\langle \sigma_1, \sigma_2 \rangle_{c_0} = (0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1) \\ (3, s_4)\end{bmatrix}} (2, (s_1, s_0)) \xrightarrow{\begin{bmatrix}(4, s_2) \\ (3, s_4)\end{bmatrix}} (3, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2) \\ (4, s_5)\end{bmatrix}} (4, (s_2, s_5)) \left(\xrightarrow{\begin{bmatrix}(5+k, s_6) \\ (5+k, s_6)\end{bmatrix}} (5+k, (s_6, s_6)) \right)_{k \geq 0}$
- $\mathrm{Cost}(\langle \sigma_1, \sigma_2 \rangle_{c_0}) = (25, 29) \rightsquigarrow \mathrm{SW}(\langle \sigma_1, \sigma_2 \rangle_{c_0}) = 54.$

Examples

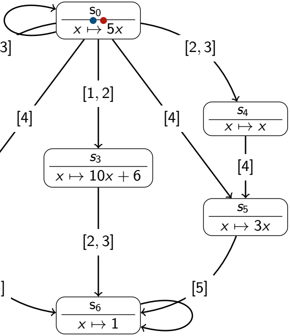


- Player 1 follows the timed path $(0, s_0)(2, s_1)(4, s_2)(5 + k, s_6)_{k \geq 0}$;
  Player 2 follows the timed path $(0, s_0)(3, s_4)(4, s_5)(5 + k, s_6)_{k \geq 0}$.

Those strategies are **blind strategies**: the players follow their timed path whatever the behavior of the other players.

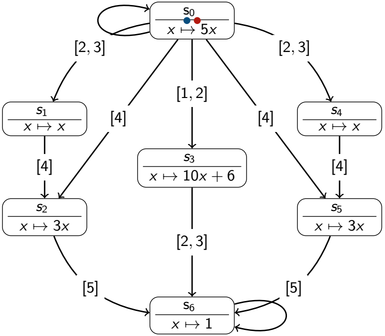
Examples

More involved strategies may be expressed



- **Player** 1 follows the timed path $(0, s_0)(1, s_3)(2 + k, s_6)_{k\geq 0}$;
  Player 2:
  - waits one time unit in s_0;
  - **observes if Player** 1 has complied with his strategy, i.e. Player 1 is in s_3;
  - yes: Player 2 follows $(2, s_4)(4, s_5)(5, s_6)$;
  - no: Player 2 follows $(4, s_2)(5, s_6)$ if Player 1 chosed the left side; $(4, s_5)(5, s_6)$ if Player 1 chosed the right side.

Nash equilibrium



Nash equilibrium

A strategy profile σ is a **Nash equilibrium** (NE) if **no** player has an **incentive to deviate unilaterally**.

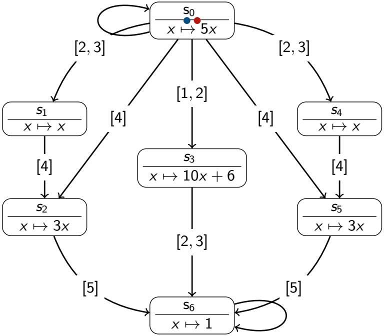
Counter-example:

- **Player** 1 follows the timed path $\rho_1 = (0, s_0)(2, s_1)(4, s_2)(5 + k, s_6)_{k \geq 0}$;
  Player 2 follows the timed path $\rho_2 = (0, s_0)(3, s_4)(4, s_5)(5 + k, s_6)_{k \geq 0}$.
- $\text{Cost}(\langle \sigma_1, \sigma_2 \rangle_{c_0}) = (25, 29)$.
- **Player 2** has an incentive to deviate and follows $\rho_2' = (0, s_0)(2, s_4)(4, s_5)(5 + k, s_6)_{k \geq 0}$

Outcome: $\langle \sigma_1, \sigma_2' \rangle_{c_0} = (0, (s_0, s_0)) \xrightarrow{\begin{bmatrix}(2, s_1)\\(2, s_4)\end{bmatrix}} (2, (s_1, s_4)) \xrightarrow{\begin{bmatrix}(4, s_2)\\(4, s_5)\end{bmatrix}} (4, (s_2, s_5)) \xrightarrow{\begin{bmatrix}(5, s_6)\\(5, s_6)\end{bmatrix}} (5, (s_6, s_6)) \dots$

Cost: $\text{Cost}_2(\langle \sigma_1, \sigma_2 \rangle_{c_0}) = 2 \cdot (5 \cdot 2) + 2 \cdot 1 + 3 \cdot 1 = \mathbf{25}$

Nash equilibrium



Nash equilibrium

A strategy profile σ is a **Nash equilibrium** (NE) if **no** player has an **incentive to deviate unilaterally**.

Existence

In all TNCGs, there exists a Nash equilibrium.

(Not explained in this talk)

Studied problems

Optimal Social Welfare - Price of Stability - Price of Anarchy

- The **optimal social welfare** (OptSW) in $(\mathcal{G}, \mathsf{c}_0)$ is

$$\mathrm{OptSW}_{\mathcal{N}} = \inf_{\sigma \in \Sigma} \mathrm{SW}(\langle \sigma \rangle_{c_0}).$$

- The **price of anarchy** (PoA) in $(\mathcal{G}, \mathsf{c}_0)$ is

$$\mathrm{PoA}_{\mathcal{N}} = \frac{\sup_{\sigma \in NE} \mathrm{SW}(\langle \sigma \rangle_{c_0})}{\mathrm{OptSW}_{\mathcal{N}}}.$$

- The **price of stability** (PoS) in $(\mathcal{G}, \mathsf{c}_0)$ is

$$\mathrm{PoS}_{\mathcal{N}} = \frac{\inf_{\sigma \in NE} \mathrm{SW}(\langle \sigma \rangle_{c_0})}{\mathrm{OptSW}_{\mathcal{N}}}$$

Studied problems

Problem 1 – Constrained social welfare

Given a TNCG $\mathcal{N}$ and a threshold $c \in \mathbb{N}$, does there exist a play ρ in $\mathcal{G}$ such that

$$\mathsf{SW}(\rho) \leq c?$$

Problem 2 – Constrained existence of a Nash equilibrium

Given a TNCG $\mathcal{N}$, a threshold vector $(x_1, \ldots, x_n) \in \mathbb{N}^n$ and a threshold $c \in \mathbb{N}$,

- does there exist a **Nash equilibrium** σ such that

$$\forall i \in N, \ \mathsf{Cost}_i(\langle \sigma \rangle_{c_0}) \leq x_i?$$

- does there exist a **Nash equilibrium** σ such that

$$\forall i \in N, \ x_i \leq \mathsf{Cost}_i(\langle \sigma \rangle_{c_0})?$$

- does there exist a **Nash equilibrium** σ such that

$$\forall i \in N, \ \mathsf{SW}(\langle \sigma \rangle_{c_0}) \leq c?$$

Studied problems

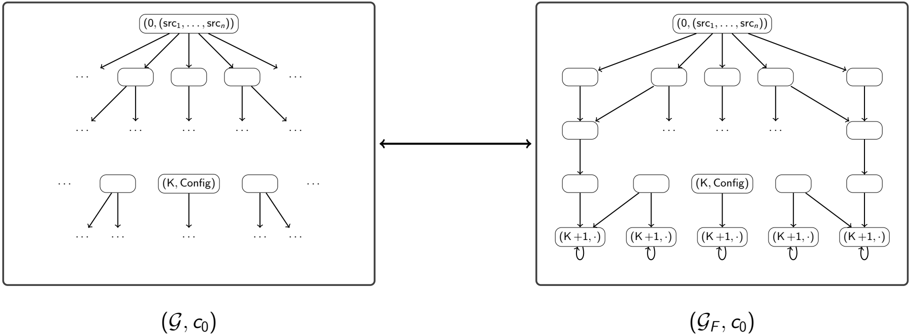
Problem 3 – Constrained price of anarchy and stability

Given a TNCG $\mathcal{N}$ and a threshold $c \in \mathbb{Q}$:

- $$\text{PoA}_{\mathcal{N}} \leq c?$$
- $$\text{PoS}_{\mathcal{N}} \leq c?$$

Constrained existence of NEs

Infinite game to finite game



Let $x \in \mathbb{N}^n$.

There exists an NE σ in $(\mathcal{G}, c_0)$ such that $\text{Cost}(\langle \sigma \rangle_{c_0}) = x$
if and only if
there exists an NE τ in $(\mathcal{G}_F, c_0)$ such that $\text{Cost}(\langle \tau \rangle_{c_0}) = x$.

Outcome characterization of NEs

Let $\mathcal{N}$ be a TNG and let $(x_1, \ldots, x_n) \in \mathbb{N}^n$,
does there exist an NE σ in $\mathcal{N}$ such that for all $i \in N$ $\text{Cost}_i(\langle\sigma\rangle_{c_0}) \leq x_i$?

- We can use approaches developped in finite concurrent games;
- In particular: **characterization of the outcome of an NE**;

Outcome characterization of a Nash equilibrium

Let ρ be a play,

there exists an NE σ such that $\langle\sigma\rangle_{c_0} = \rho$
if and only if
ρ satisfies a "good" property.

$\rightsquigarrow$ Does there exist a play ρ such that:

- for each player i, $\text{Cost}_i(\rho) \leq x_i$;
- ρ satisfies a "good" property?

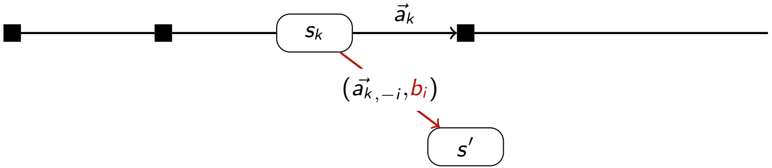
Outcome characterization of NEs

The good property

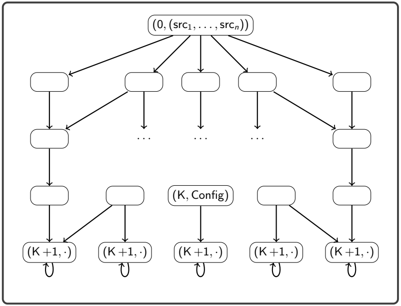
Let $\mathcal{N}$ be a TNCG and $(\mathcal{G}_F, c_0)$ be its associated finite concurrent game. A play $\rho = (s_k, \vec{a}_k, s'_k)_{k\in\mathbb{N}} \in \mathsf{Plays}_{\mathcal{G}_F}(c_0)$ is the outcome of a Nash equilibrium in $(\mathcal{G}_F, c_0)$ if, and only if,

$$\forall 1 \leq i \leq n.\ \forall k \in \mathbb{N}.\ \forall b_i \in \mathsf{Mov}_i(s_k).\ i \notin \mathsf{Visit}(\rho_{<k}) \implies \mathsf{Cost}_i(\rho_{\geq k}) \leq \underline{\mathsf{Val}}_i(s') + \mathsf{Cost}_i(s_k, (\vec{a}_{k,-i}, b_i), s')$$

where $s' = \mathsf{Up}(s_k, (\vec{a}_{k,-i}, b_i))$.



Main ideas of the algorithm



- guessing a finite path ρ in the (finite) game graph;
  - $\rightsquigarrow$ K is exponential;
  - $\rightsquigarrow$ each (d, Config) of ρ needs an exponential space to be stored;
- checking that this path satisfies the outcome characterization of NEs;
- checking that the players's cost satisfies the constraints given by the problem.

Since all $\underline{\text{Val}}_i(s)$ may be computed in EXPTIME:

> Problem 2 belongs to EXPSPACE.

Conclusion

Conclusion

- We proved that in all TNCGs there exists a Nash equilibrium (not explained in this talk – the proof relies on the notion of Potential games).
- We studied decision problems related to the qualitify of Nash equilibria in TNCGs.

	Symmetric objectives	Asymmetric objectives
Problem 1	PSPACE	EXPSPACE
Problem 2	EXPSPACE	EXPSPACE
Problem 3	EXPSPACE	EXPSPACE